Supersymmetry of Green-Schwarz superstring and matrix string theory

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We study the dynamics of a Green-Schwarz superstring on the gravitational wave background corresponding to the matrix string theory and the supersymmetry transformation rules of the superstring. The dynamics is obtained in the light-cone formulation and is shown to agree with that derived from matrix string theory. The supersymmetry structure has corrections due to the effect of the background and is identified with that of the low-energy one-loop effective action of matrix string theory in a two superstring background in the weak string coupling limit.

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I. INTRODUCTION

In the context of matrix theory, the 11-dimensional M theory in the infinite momentum frame [1] or the discrete light-cone quantized (DLCQ) M theory [2] is described nonperturbatively by the supersymmetric quantum-mechanical system with 16 supersymmetries. Upon toroidal compactification, the DLCQ M theory on a p torus with $p \leqslant 3$, has a description in terms of $(p+1)$-dimensional super Yang-Mills (SYM) theory on a dual p torus [3]. In the dual supergravity description, they can be described by M or superstring theory on the appropriate supergravity background. For the DLCQ M theory, we thus have two descriptions: the SYM theory and the supergravity theory. On the supergravity side, the process of having the DLCQ M theory and its compactified theory on a p torus, leads us to the supergravity or superstring theories in certain backgrounds, as shown in Ref. [4].

For the uncompactified case, it has been shown that the leading-order dynamics from the low-energy effective action of matrix theory, agrees well with that of the classical supergravity. (For a review, see, for example, Ref. [5] and references therein.) This remarkable agreement between these two descriptions is basically due to the fact that we have enough supersymmetry, 16 supersymmetries [6]. Though it is true that the supersymmetry alone does not give all possible dynamics of the theory, this leads us to consider an issue about to what extent the supersymmetry restricts the dynamics. Concerning this issue, one of the present authors [7] has considered the matrix theory in the supergravity side and obtained the supersymmetry transformations rules for the 11-dimensional supergraviton on the lifted $D0$-brane background, which correspond to those of the low-energy one-loop effective action of matrix theory for two supergraviton background.

In this paper, we are concerned about the matrix string theory. We study the dynamics of superstring, which is the parton of the theory, and the supersymmetry in the supergravity side. The resulting dynamics will be compared with that from the matrix string theory [8], which is the SYM description of DLCQ M theory on a circle.

Since the background corresponding to the matrix string theory is curved, as noted above, we need the superstring action in a Type IIA supergravity background for our purpose. The superstring action should be of Green-Schwarz (GS) type because the matrix string theory is the free GS light-cone superstring at its conformal point. The desired action, expanded up to quadratic order in terms of the anticommuting coordinates, has been reported in Ref. [9], which will be presented in Sec. II. In consistency checks of it, which have not been done in Ref. [9], we will show that it is supersymmetric and invariant under the κ-symmetry transformation.

The other sections are organized as follows. In Sec. III, we study the dynamics of superstring in the background corresponding to the matrix string theory and compare the results with those from the matrix string theory. The light-cone gauge is natural for our purpose, and the phase-space approach of Ref. [10] is adopted for the light-cone formulation of superstring. We would like to note that, in a recent work [11], the phase-space approach also has been well applied in the program of quantizing the GS superstring in $AdS_5 \times S^5$ [12]. In Sec. IV, we investigate the supersymmetry transformations rules, which have corrections due to the effect of the background and the supersymmetry algebra. The identification of the supersymmetry structure with that of the matrix string theory effective action for two superstring backgrounds, is discussed. Finally, a discussion follows in Sec. V.

II. GREEN-SCHWARZ SUPERSTRING ACTION

In this section, we review the ten-dimensional Type IIA GS superstring action in the bosonic supergravity backgrounds constructed in Ref. [9], with the aim of fixing our notations and for the self-containedness, and investigating its symmetries: supersymmetry and κ symmetry.

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We begin with the superstring action embedded in the ten-dimensional target superspace:[1]

$$S=\frac{1}{2\pi\alpha'}\int d^2\sigma\left(-\frac{1}{2}\sqrt{-\gamma}\gamma^{mn}\Pi_m^r\Pi_n^s\eta_{rs}+\frac{1}{2!}\epsilon^{mn}\Pi_m^A\Pi_n^B B_{BA}\right), \tag{2.1}$$

where γ_{mn} is the string worldsheet metric, and η_{rs} is the ten-dimensional flat target space-time metric. Π_m^A is the pullback of the super zehnbein $E_M{}^A$ onto the string worldsheet, with the expression

$$\Pi_m^A=\partial_m Z^M E_M{}^A, \tag{2.2}$$

and B_{AB} is the second-rank antisymmetric tensor superfield. Z^M are the supercoordinates of the target superspace and are denoted by $Z^M=(X^\mu,\theta^\alpha)$, where θ^α are the anticommuting coordinates, the 32 component Majorana spinor. (Although, using the Γ^{11} matrix, the ten-dimensional chirality operator, we can split θ into two Majorana-Weyl spinors with opposite chiralities with 16 independent components, we will keep θ to be Majorana for a while.)

The action (2.1) is the one expressed in the context of superfield formalism. For practical applications, it is less useful in its form and should be expanded in terms of anticommuting coordinates θ. The expansion coefficients and the component fields, are the functions of the ten-dimensional Type IIA supergravity fields. However, the component field expansion is a formidable task basically because of many supergravity fields: five kinds of fields even in the bosonic sector. An easy way for obtaining the expansion is given by the fact [13] that the ten-dimensional Type IIA GS superstring is related to the 11-dimensional supermembrane [14] through the double dimensional Kaluza-Klein reduction. Using this fact, the authors of Ref. [9] have constructed the Type IIA GS superstring action in the bosonic Type IIA supergravity background starting from the supermembrane action expanded up to the quadratic order in θ [15]. Though the action is not a fully expanded one and has couplings only to the bosonic backgrounds, it is enough and suitable for our purpose.

Before presenting the action, we give the component expansion of the pullback of the super zehnbein up to θ^2 order, which will be used in the discussion of symmetries and in the later sections. Following the double dimensional Kaluza-Klein reduction, we can obtain, from the expansion of super elfbein [15] with vanishing fermionic backgrounds,[2] the following expansion of the pullback of super zehnbein:[3]

$$\Pi_m^r=\partial_m X^\mu e_\mu{}^r+i\bar\theta\Gamma^r\partial_m\theta+i\partial_m X^\mu(\bar\theta\Gamma^r\Omega_\mu\theta)+\frac{i}{2}\partial_m X^\mu e_\mu{}^r(\bar\theta\Gamma^{11}\Omega_{11}\theta)+\mathcal{O}(\theta^4), \tag{2.3}$$

where we have defined

$$\Omega_\mu=\tfrac{1}{4}\omega_\mu^{rs}\Gamma_{rs}+\tfrac{1}{6}\Gamma_{rs}e^{\nu[r}e_\mu^{s]}\partial_\nu\phi+\tfrac{1}{4}\Gamma^\nu\Gamma^{11}e^\phi F_{\mu\nu}+T_\mu^{\nu\rho\sigma}\Gamma^{11}H_{\nu\rho\sigma}+T_\mu^{\nu\rho\sigma\kappa}e^\phi F'_{\nu\rho\sigma\kappa},$$

$$\Omega_{11}=-\tfrac{1}{3}\Gamma^\mu\Gamma^{11}\partial_\mu\phi-\tfrac{1}{8}\Gamma^{\mu\nu}e^\phi F_{\mu\nu}+\tfrac{1}{288}(8\Gamma^{\mu\nu\rho}H_{\mu\nu\rho}+\Gamma_{11}\Gamma^{\mu\nu\rho\sigma}e^\phi F'_{\mu\nu\rho\sigma}). \tag{2.4}$$

Here e_μ^r is the zehnbein, ω_μ^{rs} the spin connection, and ϕ the dilaton. $H_{\mu\nu\rho}$ is the field strength of the Neveu-Schwarz–Neveu-Schwarz (NS-NS) antisymmetric second-rank tensor $B_{\mu\nu}$ field. $F_{\mu\nu}$ and $F_{\mu\nu\rho\sigma}$ are the field strengths of the Ramond-Ramond (R-R) fields A_μ and $A_{\mu\nu\rho}$ related to D branes. The field strength F'_{rstu} is the modified gauge invariant four-form field strength defined by

$$F'_{\mu\nu\rho\sigma}=F_{\mu\nu\rho\sigma}+4A_{[\mu}H_{\nu\rho\sigma]}.$$

Γ^r are the ten-dimensional Dirac gamma matrices and the following tensor structures have been defined in Eq. (2.4):

$$T_\mu^{\nu\rho\sigma\lambda}\equiv\tfrac{1}{288}(\Gamma_\mu^{\nu\rho\sigma\kappa}-8\delta_\mu^{[\nu}\Gamma^{\rho\sigma\kappa]}),$$

$$T_\mu^{\nu\rho\sigma}\equiv\tfrac{1}{72}(\Gamma_\mu^{\nu\rho\sigma}-6\delta_\mu^{[\nu}\Gamma^{\rho\sigma]}),$$

where $\Gamma^{\mu\nu\cdots}$ is the totally antisymmetric products of the Dirac gamma matrices.

In writing the Type IIA GS superstring action constructed in Ref. [9], we split Majorana spinor θ into two Majorana-Weyl spinors with opposite chiralities, θ^1 and θ^2, by using the gamma matrix Γ^{11}. We assign positive chirality to θ^1 and negative chirality to θ^2;

$$\Gamma^{11}\theta^1=\theta^1,\quad \Gamma^{11}\theta^2=-\theta^2. \tag{2.5}$$

Then the Type IIA GS superstring action S in nontrivial bosonic supergravity backgrounds, expanded up to the quadratic order in the anticommuting coordinates, is

$$S=S_{\rm kin}+S_{\rm WZ}, \tag{2.6}$$

where $S_{\rm kin}$ and $S_{\rm WZ}$ are the kinetic and the Wess-Zumino (WZ) part, respectively. Their expressions are as follows:

[1]The index notations adopted here are as follows: $M,N,\ldots$ are used for the target superspace indices, while $A,B,\ldots$ are used for the tangent superspace. As usual, a superspace index is the composition of two types of indices such as $M=(\mu,\alpha)$ and $A=(r,a)$. $\mu,\nu,\ldots$ $(r,s,\ldots)$ are the ten-dimensional target (tangent) space-time indices taking values in $0,1,\ldots,9$. $\alpha,\beta,\ldots$ $(a,b,\ldots)$ are the ten-dimensional (tangent) spinor indices with values in $1,2,\ldots,32$. $m,n,\ldots$ are the worldsheet vector indices with values τ and σ. The convention for the worldsheet antisymmetric tensor is taken to be $\epsilon^{\tau\sigma}=1$.

[2]For the vanishing fermionic backgrounds as in this paper, the order of θ increases by two for all quantities having expansion in terms of θ; if the leading-order term is of the even (odd) order in θ, all the higher-order terms are of the even (odd) order in θ.

[3]In our convention, $\bar\theta=\theta^{\rm T}\Gamma^0$.

$$S_{\rm kin}=-\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-\gamma}\gamma^{mn}(\partial_m X^\mu+i\bar{\theta}^I\Gamma^\mu\partial_m\theta^I)(\partial_n X^\nu+i\bar{\theta}^J\Gamma^\nu\partial_n\theta^J)G_{\mu\nu}-\frac{i}{4\pi\alpha'}\int d^2\sigma\sqrt{-\gamma}\gamma^{mn}\partial_m X^\mu\partial_n X^\nu\times\Big[\frac{1}{2}(\bar{\theta}^I\Gamma_{\mu rs}\theta^I)\omega_\nu^{rs}-\frac{1}{4}s^{IJ}(\bar{\theta}^I\Gamma_\mu^{\rho\sigma}\theta^J)H_{\nu\rho\sigma}+\frac{1}{4}(\bar{\theta}^1\Gamma^{\rho\sigma}\theta^2)e^\phi F_{\rho\sigma}G_{\mu\nu}-(\bar{\theta}^1\Gamma_\mu^\rho\theta^2)e^\phi F_{\nu\rho}+\frac{1}{48}(\bar{\theta}^1\Gamma^{\rho\sigma\kappa\lambda}\theta^2)e^\phi F'_{\rho\sigma\kappa\lambda}G_{\mu\nu}-\frac{1}{6}(\bar{\theta}^1\Gamma_\mu^{\rho\sigma\kappa}\theta^2)e^\phi F'_{\nu\rho\sigma\kappa}+\mathcal{O}(\theta^4)\Big], \tag{2.7}$$

where $G_{\mu\nu}$ is the target space-time metric, $I,J=1,2$, and s^{IJ} is defined as

$$s^{11}=-s^{22}=1,\quad s^{12}=s^{21}=0,$$

and

$$S_{\rm WZ}=-\frac{i}{2\pi\alpha'}\int d^2\sigma\epsilon^{mn}s^{IJ}\Big(\partial_m X^\mu+\frac{i}{2}\bar{\theta}^K\Gamma^\mu\partial_m\theta^K\Big)\times(\bar{\theta}^I\Gamma_\mu\partial_n\theta^J)-\frac{1}{4\pi\alpha'}\int d^2\sigma\epsilon^{mn}\partial_m X^\mu\partial_n X^\nu\times\Big[B_{\mu\nu}+\frac{i}{2}s^{IJ}(\bar{\theta}^I\Gamma_{\mu rs}\theta^J)\omega_\nu^{rs}-\frac{i}{4}(\bar{\theta}^I\Gamma_\mu^{\rho\sigma}\theta^I)H_{\nu\rho\sigma}+\frac{i}{2}(\bar{\theta}^1\theta^2)e^\phi F_{\mu\nu}+\frac{i}{4}(\bar{\theta}^1\Gamma_{\mu\nu}^{\rho\sigma}\theta^2)e^\phi F_{\rho\sigma}+\frac{i}{4}(\bar{\theta}^1\Gamma^{\rho\sigma}\theta^2)e^\phi F'_{\mu\nu\rho\sigma}+\frac{i}{48}(\bar{\theta}^1\Gamma_{\mu\nu}^{\rho\sigma\kappa\lambda}\theta^2)\times e^\phi F'_{\rho\sigma\kappa\lambda}+\mathcal{O}(\theta^4)\Big]. \tag{2.8}$$

There are several notable features in these actions. First of all, as expected, we have explicit e^ϕ coupling in the linear terms in the R-R fields, A_μ and $A_{\mu\nu\rho}$, which was first suggested by Tseytlin [16]. Furthermore, as fundamental strings are neutral under R-R fields, it is natural to couple with R-R fields, if any, via their field strength. In particular, considering these actions as describing the interactions of the fundamental string with the background supergravity fields, these couplings have a natural interpretation as a spin-orbitlike coupling with background R-R fields and imply that the fundamental string has dipole interactions with R-R fields. Indeed the interactions with the Lorentz spin connection give the genuine spin-orbit coupling between the string and the gravitational backgrounds and has been extensively studied with regard to the matrix theory [6,17–20].

Note also that, at the linearized level in the supergravity fields $B_{\mu\nu}$, A_μ, $A_{\mu\nu\rho}$, $g_{\mu\nu}$, and φ, where $G_{\mu\nu}=\eta_{\mu\nu}+g_{\mu\nu}$ and $\phi=\phi_\infty+\varphi$, the action can be thought of as the sum of free string action and vertex operators for emission of bosonic supergravity fields.[4] In this sense it is very natural that the vertex operators for R-R fields starts with θ^2 order, as there are target space-time supersymmetries connecting the NS sector and R sector. Schematically, under space-time supersymmetry transformations in their leading order in θ, the vertex operators $V_{\text{NS-NS}}$ and $V_{\text{R-R}}$ corresponding to the fields in the NS-NS and R-R sectors, respectively, should satisfy

$$\delta_B V_{\text{NS-NS}}+\delta_F V_{\text{R-R}}=0,$$

where subscripts B and F denote the supersymmetry transformation of bosonic coordinate X^μ and fermionic coordinate θ, respectively. In the following section, we will see that this is indeed the case.

A. Local supersymmetry

In this and the next section, we investigate the invariance of the superstring action S, Eq. (2.6), under the supersymmetry and κ-symmetry transformation in order to check whether or not the action was correctly expanded. We note that, for the simplicity of presentation, we will not split Majorana spinors (θ,η,κ) into Majorana-Weyl spinors.

The local supersymmetry, super diffeomorphism, is the local change of supercoordinates $(\delta_\eta X^\mu,\delta_\eta\theta^\alpha)$. Since its parameters are superfields, the transformations of supercoordinates have component expansion in terms of θ. In the 11-dimensional case, the expansions of the parameters have been given in Ref. [15]. With the vanishing fermionic backgrounds, the Kaluza-Klein reduction of them to ten dimensions leads to

$$\delta_\eta X^\mu=i\bar{\theta}\Gamma^\mu\eta+\mathcal{O}(\theta^3),$$
$$\delta_\eta\theta=\eta+\mathcal{O}(\theta^2). \tag{2.9}$$

The terms on the right-hand sides are enough for the transformation of the superstring action expanded up to θ^2 order, since higher-order corrections in the transformation rules require terms of higher order than θ^2 order in the action. Furthermore, the supersymmetry variation of the action is valid up to the linear order in θ, because the transformation $\delta_\eta X^\mu$ acting on the terms of θ^2 order, requires the θ^3 order terms in the action.

The superstring action contains background fields as well as supercoordinates. Thus the action is in fact not invariant with only the above transformations, Eq. (2.9). As noted by the authors of Ref. [15], the invariance of the action means that the super diffeomorphism induces the supersymmetry transformations of the background fields. In other words, the action is supersymmetric if its variation under the super dif-

[4]Recent construction of vertex operators in the GS superstring theory is given in Refs. [21,22].

feomorphism vanishes modulo supersymmetry transformations of the background fields. This fact requires us to have the supersymmetry transformation rules for the background fields. What we need are rules for fermion background fields, because the transformation rules of bosonic fields lead to fermionic fields, which are turned off in this paper. Fermion fields in ten-dimensional supergravity are the gravitino ψ_μ and the dilatino, λ, which are Majorana fermions and split into two Majorana-Weyl fermions with opposite chiralities $\psi_\mu^{1,2}$ and $\lambda^{1,2}$, respectively. We note that their transformation rules are to be written in the string frame, since the object affected by the backgrounds is the string. Under the supersymmetry variation, the gravitino transforms as

$$\begin{aligned}\delta_\eta\psi_\mu = & D_\mu(\omega)\,\eta - \tfrac{1}{8}\Gamma_r\Gamma_s\,\eta e_\mu^r e^{\nu s}\partial_\nu\phi - \tfrac{1}{64}e_\mu^r\\ &\times(\Gamma_{rst}-14\eta_{rs}\Gamma_t)\Gamma^{11}\eta e^{\phi}F^{st} + \tfrac{1}{96}e_\mu^r\\ &\times(\Gamma_r^{stu}-9\delta_r^s\Gamma^{tu})\Gamma^{11}\eta H_{stu} + \tfrac{1}{768}e_\mu^r\\ &\times(3\Gamma_r^{stuv}-20\delta_r^s\Gamma^{tuv})\,\eta e^{\phi}F'_{stuv}, \end{aligned} \tag{2.10}$$

where the Lorentz covariant derivative $D_\mu(\omega)$, is given by $D_\mu(\omega)=\partial_\mu+\frac{1}{4}\omega_\mu^{rs}\Gamma_{rs}$. As for the dilatino field, the supersymmetry transformation rule is

$$\begin{aligned}\delta_\eta\lambda = & -\frac{1}{2\sqrt{2}}\Gamma^r\Gamma^{11}\eta e_r^\nu\partial_\nu\phi - \frac{3}{16\sqrt{2}}\Gamma^{rs}\eta e^{\phi}F_{rs}\\ &+\frac{1}{24\sqrt{2}}\Gamma^{rst}\eta H_{rst} + \frac{1}{192\sqrt{2}}\Gamma^{rstu}\Gamma^{11}\eta e^{\phi}F'_{rstu}. \end{aligned} \tag{2.11}$$

These transformation rules are those with the vanishing fermion backgrounds. In the study of supergravity, the transformation rules are usually written in the Einstein frame [23], which can be obtained from the above transformation rules by a suitable rescaling of fields and supersymmetry parameter. The resulting transformation rules are the same as above except for the absence of the term involving the derivative of the dilaton in Eq. (2.10) and some change in powers of the dilaton factor.

For the supersymmetry variation of the kinetic term, Eq. (2.7), it is enough to consider the variation of the pullback of superzehnbein under the super diffeomorphism, Eq. (2.9). A straightforward calculation shows that

$$\begin{aligned}\delta_\eta\Pi_m^r = & i\partial_m X^\mu\left(2\bar{\theta}\Gamma^r\delta_\eta\psi_\mu + \frac{1}{\sqrt{2}}e_\mu^r\bar{\theta}\Gamma^{11}\delta_\eta\lambda\right)\\ &-\Lambda_s^r\Pi_m^s + \mathcal{O}(\theta^3), \end{aligned} \tag{2.12}$$

where Λ_s^r is the local Lorentz transformation parameter associated with the super diffeomorphism and is precisely given by

$$\begin{aligned}\Lambda^{rs} = & i(\bar{\theta}\Gamma^\mu\eta)\,\omega_\mu^{rs} - \frac{1}{3}i\bar{\theta}(\Gamma^{rs}\Gamma^t-\Gamma^{rst})\,\eta e_t^\mu\partial_\mu\phi\\ &+\frac{1}{3\sqrt{2}}i\bar{\theta}\Gamma^{rs}\Gamma_{11}\delta_\eta\lambda - \frac{1}{2}i(\bar{\theta}\Gamma_{11}\eta)e^{\phi}F^{rs}\\ &+\frac{1}{36}i\bar{\theta}(\Gamma^{rstuv}\Gamma^{11}H_{tuv}+12\Gamma_t\Gamma_{11}H^{rst})\,\eta\\ &+\frac{1}{144}i\bar{\theta}(\Gamma^{rstuvw}e^{\phi}F'_{tuvw}+24\Gamma_{tu}e^{\phi}F'^{rstu})\,\eta + \mathcal{O}(\theta^3). \end{aligned} \tag{2.13}$$

Though it has a slightly complicated expression, Λ_s^r does not enter into the variation of the kinetic part, since two superzehnbeins enter into the kinetic part symmetrically. We now see the supersymmetry transformations of fermion backgrounds, $\delta_\eta\psi_\mu$ and $\delta_\eta\lambda$, which have been identified on the right-hand sides of Eqs. (2.10) and (2.11) at the final stage of deriving Eqs. (2.12) and (2.13). However, this does not conclude that the component expansion of the pullback of superzehnbein or the kinetic part has been obtained correctly, since the expansion has been given with the vanishing fermion backgrounds. Thus, we turn on the fermion backgrounds temporarily and investigate how they appear in the superzehnbein. What we should be concerned about are the linear order terms in θ, which can be seen by looking at the superelfbein expanded in Ref. [15]. The corresponding term in the expansion of the superelfbein is $2i\bar{\theta}\Gamma^{\hat{r}}\hat{\psi}_{\hat{\mu}}$, where $\hat{\psi}$ is the 11-dimensional gravitino, and $\hat{r}$ and $\hat{\mu}$ are the 11-dimensional flat and curved indices, respectively. The Kaluza-Klein dimensional reduction of it can be done in the usual manner,[5] and leads to

$$2i\bar{\theta}\Gamma^r\psi_\mu + \frac{i}{\sqrt{2}}\bar{\theta}\Gamma^{11}\lambda e_\mu^r - \frac{i}{3\sqrt{2}}\bar{\theta}\Gamma_\mu^r\Gamma^{11}\lambda. \tag{2.14}$$

We see that the supersymmetry variation of the fermion backgrounds in this result exactly matches with the terms appearing in Eqs. (2.12) and (2.13). This concludes that the kinetic term of the superstring action has the correct component expansion and is consistent with the local supersymmetry.

We now turn to the supersymmetry variation of the WZ term, Eq. (2.8). After the same type of calculations with those for the kinetic term, it is given by

[5]Through the Kaluza-Klein reduction, the 11-dimensional gravitino is related to the 10-dimensional gravitino ψ_μ, the dilatino λ, and the dilaton ϕ as follows: $\hat{\psi}_\mu = e^{-\phi/6}(\psi_\mu - (\sqrt{2}/12)\Gamma_\mu\Gamma^{11}\lambda)$, $\hat{\psi}_{11}=(2\sqrt{2}/3)e^{5\phi/6}\lambda$. Here the ten-dimensional quantities are those in the string frame.

$$\delta_\eta\left(\frac{1}{2!}\epsilon^{mn}\Pi_m^A\Pi_n^B B_{BA}\right)$$
$$=-i\epsilon^{mn}\partial_m X^\mu\partial_n X^\nu\left(2\bar{\theta}\Gamma^{11}\Gamma_\nu\delta_\eta\psi_\mu+\frac{1}{\sqrt{2}}\bar{\theta}\Gamma_{\nu\mu}\delta_\eta\lambda\right)$$
$$+\partial_m S^m+\mathcal{O}(\theta^3), \tag{2.15}$$

where $\partial_m S^m$ is a surface term, which can be ignored since we are concerned about the closed string. This is the desired result. However, as in the case of the kinetic term, we need to know the terms of linear order in θ containing the fermion backgrounds in order to check the supersymmetry of the WZ term. The 11-dimensional term relevant to them comes from the component expansion of the third-rank tensor superfield, and is given by $-6i\bar{\theta}\Gamma_{[\widehat{11}\mu}\hat{\psi}_{\nu]}$ [15]. Through the dimensional reduction, it reduces to

$$-4i\bar{\theta}\Gamma_{11}\Gamma_{[\mu}\psi_{\nu]}-\sqrt{2}i\bar{\theta}\Gamma_{\mu\nu}\lambda. \tag{2.16}$$

Obviously, the supersymmetry variation of this exactly matches with the terms of Eq. (2.15). This tells us that the WZ term of the superstring action as well as the kinetic term has the correct component expansion and is consistent with the local supersymmetry.

We have seen that each term of the superstring action transforms properly under the supersymmetry transformation, and has the consistent and correct component expansion.

B. κ symmetry

As another consistency check, we now consider the invariance of the superstring action, Eq. (2.6), under the κ transformation. The investigation of the κ symmetry will give us the confirmation on the correctness of the relative coefficient between the kinetic and the WZ part.

For the κ symmetry, it is convenient to work with the superstring action with Nambu-Goto type rather than the Polyakov type action, Eq. (2.1), which avoids the complexity due to the variation of the worldsheet metric γ^{mn}. The Nambu-Goto-type action is obtained by solving the classical equation of motion for γ^{mn} and putting the result back into the action Eq. (2.1), and is as follows:

$$S_{\rm NG}=\frac{1}{2\pi\alpha'}\int d^2\sigma\left(-\sqrt{-g}+\frac{1}{2!}\epsilon^{ij}\Pi_i^A\Pi_j^B B_{BA}\right), \tag{2.17}$$

where g is the determinant of the induced metric g_{mn} given by

$$g_{mn}=\Pi_m^r\Pi_n^s\eta_{rs}. \tag{2.18}$$

We notice that the κ transformation rules also have a component expansion in terms of θ. As in the case of supersymmetry, however, only the leading-order terms in the expansion are needed to show that the superstring action, constructed up to θ^2 order, is κ symmetric:

$$\delta_\kappa X^\mu=i\bar{\kappa}_+\Gamma^\mu\theta+\mathcal{O}(\theta^3),$$
$$\delta_\kappa\theta^a=\kappa_+^a+\mathcal{O}(\theta^2), \tag{2.19}$$

where we have defined

$$\kappa_+=(1+\Gamma\Gamma^{11})\kappa. \tag{2.20}$$

The matrix Γ is given by

$$\Gamma=\frac{1}{2\sqrt{-g}}\epsilon^{mn}\Pi_m^r\Pi_n^s\Gamma_r\Gamma_s=\frac{1}{2\sqrt{-g}}\epsilon^{mn}\Gamma_m\Gamma_n, \tag{2.21}$$

where Γ_m are the pullback onto the worldvolume of the space-time Dirac gamma matrices:

$$\Gamma_m=\Pi_m^r\Gamma_r. \tag{2.22}$$

Γ has the properties such as projection operator and anticommutes with the pulled back gamma matrices Γ_m:

$$\Gamma^2=1,\quad {\rm Tr}\,\Gamma=0,$$
$$\Gamma\Gamma^m=-\Gamma^m\Gamma=-\frac{1}{\sqrt{-g}}\epsilon^{mn}\Gamma_n. \tag{2.23}$$

Under the κ transformations, Eq. (2.19), the pullback of the superzehnbein transforms as

$$\delta_\kappa\Pi_i^r=2i\bar{\kappa}_+\Gamma^r\partial_i\theta+i\Pi_i^s(2\bar{\kappa}_+\Gamma^r\tilde{\Omega}_s\theta+\bar{\kappa}_+\Gamma^{11}\tilde{\Omega}_{11}\theta\delta_s^r)$$
$$+M_s^r\Pi_i^s+\mathcal{O}(\theta^3), \tag{2.24}$$

where $\tilde{\Omega}_s(\tilde{\Omega}_{11})$ is $\Omega_s(\Omega_{11})$ in Eq. (2.4) without terms involving the derivatives of dilaton ϕ. M_s^r is an antisymmetric matrix just like the Lorentz transformation parameter Λ_s^r in Eq. (2.12), whose detailed from is not necessary because it does not give any contribution to the variation of the superstring action. The κ transformation of the kinetic term of the Nambu-Goto type is then

$$\delta_\kappa S_{\rm kin}=-\frac{i}{2\pi\alpha'}\int d^2\sigma\sqrt{-g}g^{mn}[2\Pi_m^r\bar{\kappa}_+\Gamma_r\partial_n\theta$$
$$+\Pi_m^r\Pi_n^s(2\bar{\kappa}_+\Gamma_r\tilde{\Omega}_s\theta+\eta_{rs}\bar{\kappa}_+\Gamma^{11}\tilde{\Omega}_{11}\theta)+\mathcal{O}(\theta^3)]. \tag{2.25}$$

For the WZ term, we have the following κ transformation:

$$\delta_\kappa S_{\rm WZ}=\frac{i}{2\pi\alpha'}\int d^2\sigma\epsilon^{mn}[-2\Pi_m^r\bar{\kappa}_+\Gamma_r\Gamma^{11}\partial_n\theta$$
$$+\Pi_m^r\Pi_n^s(2\bar{\kappa}_+\Gamma^{11}\Gamma_r\tilde{\Omega}_s\theta+\bar{\kappa}_+\Gamma_{rs}\tilde{\Omega}_{11}\theta)+\mathcal{O}(\theta^3)]. \tag{2.26}$$

With these transformations, and by using some properties of the matrix Γ, Eq. (2.23), we can now show that the Nambu-Goto type string action, Eq. (2.17), is κ symmetric up to the quadratic order in θ:

$$\delta_\kappa S_{\rm NG} = -\frac{i}{\pi\alpha'}\int d^2\sigma\sqrt{-g}\,\overline{\kappa_+}(1-\Gamma\Gamma^{11})(\Gamma^m\partial_m+\Gamma^m\tilde{\Omega}_m + \Gamma^{11}\tilde{\Omega}_{11})\theta + \mathcal{O}(\theta^3) = 0+\mathcal{O}(\theta^3), \tag{2.27}$$

where $\overline{\kappa_+}=\overline{\kappa}(1+\Gamma\Gamma^{11})$ and $(1+\Gamma\Gamma^{11})(1-\Gamma\Gamma^{11})=0$ have been used.

III. GS SUPERSTRING ON THE GRAVITATIONAL WAVE BACKGROUND

In this section, by using the superstring action Eq. (2.6) presented in the previous section, we study the dynamics of the superstring on the gravitational wave background corresponding to the matrix string theory. From the matrix string theory side calculations, though the dynamics related to the fermion bilinears has not been completely determined, we compare some of the reported results with ours [24]. As will be shown in this section, they agree with the bosonic part of our result.

The ten-dimensional supergravity background, corresponding to the matrix string theory [4] is the geometry obtained after the following procedure; one begins with the $D0$-brane geometry, follows the prescription of Seiberg and Sen [3] and with the TST-duality chain the same as that used for obtaining the matrix string theory [8]. In the resulting background, only the metric is nontrivial. The dilaton is just constant and all other supergravity fields are simply zero. If we introduce the light-cone coordinates

$$x^\pm = x^9 \pm x^0, \tag{3.1}$$

then the background geometry is as follows:

$$ds^2 = dx^+dx^- + h(dx^-)^2 + (dx^i)^2,$$
$$e^\phi = g_s, \tag{3.2}$$

where i is the index for the eight-dimensional flat transverse space taking values in $1,\ldots,8$ and g_s as the string coupling constant. As an important structure of this geometry, the lightlike direction x^- is compactified with the radius R, leading to the DLCQ:

$$x^- \sim x^- + 2\pi nR, \quad n\in\mathbf{Z}. \tag{3.3}$$

h is the harmonic function in the eight-dimensional transverse space spanned by x^i and with $r=(x^ix^i)^{1/2}$ is given by

$$h = \frac{4}{\pi}\frac{g_s^2 l_s^8}{R^2}\frac{N_s}{r^6}, \tag{3.4}$$

where l_s is the string scale and N_s the light-cone momentum of the background. In addition to the above background geometry, what we need to write down the superstring action is the spin connection for the geometry, whose nonvanishing components are given by

$$\omega_i^{\,0} = \omega_0^{\,i} = \omega_i^{\,9} = -\omega_9^{\,i} = \tfrac{1}{2} f^{-1/2}\partial_i h\, dx^-,$$
$$\omega_9^{\,0} = \omega_0^{\,9} = \tfrac{1}{2} f^{-1}\partial_i h\, dx^i, \tag{3.5}$$

where we have defined $f=1+h$.

As is well known and shown explicitly in the previous section, the GS superstring action has worldsheet local fermionic κ symmetry, which indicates the doubling of the degrees of freedom described by θ. Our study of the string dynamics begins with the consideration of fixing the fermionic symmetry. Since our basic concern is the supergravity side description of the DLCQ M theory, the light-cone gauge fixing condition is very natural. In the flat background, the light-cone gauge fixing condition for the κ symmetry enables us to have a greatly simplified action. As we shall see, this is also the case in the background (3.2). The κ-symmetry fixing condition we choose is then

$$\Gamma^+\theta^I = 0. \tag{3.6}$$

In order to solve this, the representation for the $SO(1,9)$ Dirac gamma matrices is in order. The representation we take in this paper is as follows:

$$\Gamma^0 = i\sigma^2\otimes\mathbf{1}_{16}, \quad \Gamma^9=\sigma^3\otimes\mathbf{1}_{16}, \quad \Gamma^i = \sigma^1\otimes\gamma^i,$$
$$\Gamma^\pm = \Gamma^9 \pm \Gamma^0, \tag{3.7}$$

where σ's are Pauli matrices, and $\mathbf{1}_{16}$ is the 16×16 unit matrix. γ^i are the 16×16 symmetric real gamma matrices satisfying the spin (8) Clifford algebra $\{\gamma^i,\gamma^j\}=2\delta^{ij}$, which are actually reducible to the $\mathbf{8_s}+\mathbf{8_c}$ representation of spin (8). With this representation, the κ-symmetry fixing condition (3.6) implies that θ^I has the following form:

$$\theta^I = \begin{pmatrix}\psi^I\\ -\psi^I\end{pmatrix} \tag{3.8}$$

where ψ^I is the 16-component Majorana-Weyl spinor.

The gauge condition for the κ symmetry now simplifies the superstring action in the background geometry (3.2) as

$$S = -\frac{1}{2}\int d^2\sigma\sqrt{-\gamma}[\gamma^{mn}(\partial_m X^+ + h\partial_m X^-)\partial_n X^- + \gamma^{mn}\partial_m X^i\partial_n X^i + 8if^{-1/2}(\partial_m X^+ + h\partial_m X^-)\times P^{mn,IJ}(\psi^I\partial_n\psi^J) + 4if^{-1/2}\partial_i h\partial_m X^j\partial_n X^- P^{mn,IJ}\times(\psi^I\gamma^{ij}\psi^J) + \mathcal{O}(\psi^4)], \tag{3.9}$$

where Eq. (3.8) has been used and $P^{mn,IJ}$ is defined by

$$P^{mn,IJ} \equiv \frac{1}{2}\left(\gamma^{mn}\delta^{IJ} + \frac{\epsilon^{mn}}{\sqrt{-\gamma}}s^{IJ}\right), \tag{3.10}$$

which are the projection tensors that project a worldsheet vector into its self-dual ($I=J=1$) or anti-self-dual ($I=J=2$) pieces. For notational convenience, we have set $2\pi\alpha'=1$ in the action (3.9). We note the overall factor $f^{-1/2}$ in the fermion bilinear terms can be made disappear in the action simply via the rescaling

$$\psi^I \to f^{1/4}\psi^I. \tag{3.11}$$

This is possible due to the fact that the Majorana-Weyl spinors ψ^I satisfy $\psi^I\psi^I=0$. In what follows, this rescaling will be understood.

Having fixed the κ symmetry, the action (3.9) still has an additional local symmetry, the worldsheet diffeomorphism. We fix this symmetry by taking the light-cone gauge. Since the diffeomorphism is two parameter symmetry, two conditions are required. We choose the first to be that the light-cone time X^+ is proportional to the worldsheet time τ. In the DLCQ framework, since a certain sector of the light-cone momentum, canonical momentum of X^-, is considered, it is convenient to have constant light-cone momentum independent on the worldvolume spatial coordinate σ. Thus it is natural to choose the second condition as constant light-cone momentum. This type of light-cone gauge, the same type as in the case of a flat background, is the one taken in what is known as the phase-space approach of string quantization [10]. The phase space means that we should formulate our system in its phase space because one of the gauge fixing conditions we choose is imposed on a canonical momentum. In order to impose our gauge fixing condition, we should rewrite the Lagrangian in the phase space, that is, in the first-order form. It should be noted here that we will not touch the fermionic part, since the canonical momentum of the fermionic coordinate is a constraint, which will be treated later. We first obtain the canonical momenta of bosonic coordinates from

$$P^+=\frac{\partial\mathcal{L}}{\partial\dot{X}^-},\quad P^-=\frac{\partial\mathcal{L}}{\partial\dot{X}^+},\quad P^i=\frac{\partial\mathcal{L}}{\partial\dot{X}^i},$$

where the dot means the derivative with respect to the worldsheet time τ. In what follows, the prime will be used for the derivative with respect to the worldsheet spatial coordinate σ. The explicit expressions for the canonical momenta are then as follows:

$$\begin{aligned}P^+=&-\tfrac{1}{2}\sqrt{-\gamma}[\gamma^{\tau\tau}(\dot{X}^+ + 2h\dot{X}^-)+\gamma^{\tau\sigma}(X'^+ + 2hX'^-)\\&+8ihP^{\tau m,IJ}(\psi^I\partial_m\psi^J)+4i\partial_i h\partial_m X^j P^{m\tau,IJ}(\psi^I\gamma^{ij}\psi^J)]\\&+\mathcal{O}(\psi^4),\\P^-=&-\tfrac{1}{2}\sqrt{-\gamma}[\gamma^{\tau\tau}\dot{X}^- + \gamma^{\tau\sigma}X'^- + 8iP^{\tau m,IJ}(\psi^I\partial_m\psi^J)]\\&+\mathcal{O}(\psi^4),\\P^i=&-\sqrt{-\gamma}[\gamma^{\tau\tau}\dot{X}^i+\gamma^{\tau\sigma}X'^i\\&-2i\partial_j h\partial_m X^- P^{\tau m,IJ}(\psi^I\gamma^{ij}\psi^J)]+\mathcal{O}(\psi^4).\end{aligned}\tag{3.12}$$

From the action (3.9), the Lagrangian in the first-order form is then given by

$$\begin{aligned}\mathcal{L}=&\dot{X}^+P^- + \dot{X}^-P^+ + \dot{X}^iP^i + 4i\Big((P^+ - hP^-)\delta^{IJ}\\&+\frac{1}{2}(X'^+ + hX'^-)s^{IJ}\Big)(\psi^I\dot{\psi}^J)+\frac{2}{\gamma^{\tau\tau}\sqrt{-\gamma}}\mathcal{H}_{00}+\frac{\gamma^{\tau\sigma}}{\gamma^{\tau\tau}}\mathcal{H}_{01},\end{aligned}\tag{3.13}$$

where

$$\begin{aligned}\mathcal{H}_{00}=&P^-(P^+ - hP^-)+\tfrac{1}{4}(P^i)^2+\frac{1}{4}[X'^-(X'^+ + hX'^-)\\&+(X'^i)^2]+2i((P^+ - hP^-)s^{IJ}+\tfrac{1}{2}(X'^+ + hX'^-)\delta^{IJ})\\&\times(\psi^I\psi'^J)-i\partial_i h(P^-(P^j\delta^{IJ}+X'^j s^{IJ})\\&-\tfrac{1}{2}X'^-(P^j s^{IJ}+X'^j\delta^{IJ}))(\psi^I\gamma^{ij}\psi^J)+\mathcal{O}(\psi^4),\end{aligned}\tag{3.14}$$

$$\begin{aligned}\mathcal{H}_{01}=&P^-X'^+ + P^+X'^- + P^iX'^i + i((P^+ - hP^-)\delta^{IJ}\\&+\tfrac{1}{2}(X'^+ + hX'^-)s^{IJ})(\psi^I\psi'^J)+\mathcal{O}(\psi^4).\end{aligned}\tag{3.15}$$

In Eq. (3.13), the worldsheet metric is nondynamical and its combinations, $1/(\gamma^{\tau\tau}\sqrt{-\gamma})$ and $\gamma^{\tau\sigma}/\gamma^{\tau\tau}$, act as Lagrange multiplier fields leading to the constraints

$$\mathcal{H}_{00}\approx 0,\quad \mathcal{H}_{01}\approx 0, \tag{3.16}$$

which are just the Virasoro constraints. As usual, these enable us to determine P^- and X'^- in terms of the canonical pairs of the transverse coordinates, (X^i,P^i), and the fermionic coordinates ψ^I.

We now fix the worldsheet diffeomorphism by the light-cone gauge alluded to above, which is explicitly given by

$$X^+=2\tau,\quad P^+=p^+l^{-1}=\text{const} \tag{3.17}$$

where l is the range of σ integration, which we set equal to one ($l=1$). p^+ is the center of mass momentum in the X^- direction and its value is quantized as $p^+=N/R$ (N is an integer and means that we are in the N sector of DLCQ) since the X^- direction is compactified as in Eq. (3.3). After imposing this light-cone gauge and the constraints [Eq. (3.16)] in a strong sense, we are left with the reduced system containing only the physical degrees of freedom; (X^i,P^i), ψ^I and the canonical pair corresponding to the center-of-mass mode of X^-, (x^-,p^+).

In the phase space, the dynamics are described by the canonical Hamiltonian. For our light-cone gauge fixed system, it is just the light-cone Hamiltonian and is given by

$$H=\dot{x}^-p^+ + \int_0^1 d\sigma(\dot{X}^iP^i + \dot{\psi}^IP^I - \mathcal{L}) = -2\int_0^1 d\sigma P^-, \tag{3.18}$$

where P^I is the canonical momentum of ψ^I given by

$$P^I = \frac{\partial \mathcal{L}}{\partial \dot{\psi}^I} = -4i\left((p^+ - hP^-)\delta^{IJ} + \frac{1}{2}hX'^- s^{IJ} \right)\psi^J + \mathcal{O}(\psi^3). \tag{3.19}$$

The detailed form of the Hamiltonian is then

$$\begin{aligned} H = \frac{1}{2p^+}\int_0^1 d\sigma \Bigg\{ & (P^i)^2 + (X'^i)^2 + i\psi^1\psi'^1 - i\psi^2\psi'^2 \\ & - \frac{h}{4(p^+)^2}(P^i + X'^i)^2(P^j - X'^j)^2 - \frac{ih}{4(p^+)^2} \\ & \times (P^i - X'^i)^2\psi^1\psi'^1 + \frac{i}{8(p^+)^2}\partial_i h(P^k - X'^k)^2 \\ & \times (P^j + X'^j)\psi^1\gamma_{ij}\psi^1 + \frac{ih}{4(p^+)^2}(P^i + X'^i)^2\psi^2\psi'^2 \\ & + \frac{i}{8(p^+)^2}\partial_i h(P^k + X'^k)^2(P^j - X'^j)\psi^2\gamma_{ij}\psi^2 \\ & + \cdots + \mathcal{O}(\psi^4)\Bigg\}, \end{aligned} \tag{3.20}$$

where, in order to make the kinetic term for ψ^I to be of the canonical form, the following rescaling has been performed:

$$\psi^I \to \frac{1}{2\sqrt{2p^+}}\psi^I. \tag{3.21}$$

The dots in Eq. (3.20) denote the terms of order $\mathcal{O}(h^2)$, which basically correspond to those of higher derivatives than four. These are beyond our interest, since in the supergravity side analysis, we are concerned about the terms corresponding to the so-called one-loop exact F^4 or four-derivative terms and their superpartners in the low-energy effective action from the SYM side, which are of linear order in h in Eq. (3.20). Thus, from now on, we will keep only the terms up to linear order in h in all expressions in the remaining part of this paper.

The Hamiltonian (3.20) shows typical interaction terms between the superstring and the background geometry (3.2). The bosonic interaction is the spinless one, and Eq. (3.4) tells us that it has $1/r^6$ behavior. In the SYM side, this type of behavior can be seen in the perturbative sector of the result of Ref. [25]. Comparing the result of Ref. [24], the structure of the interaction also agrees with that in the SYM side; more precisely, the matrix string stress tensor T^{--} in the weak string coupling limit in that reference. The term proportional to $\psi^I\gamma^{ij}\psi^I$ is the spin-orbit interaction term, which has been found and studied also in other compactifications of DLCQ M theory [19,20].

Before closing this section, we obtain the equations of motion for X^i and ψ^I by using the Hamiltonian (3.20) as the time evolution operator, which will be used in the next section. By the way, since we have constraints coming from the momenta of ψ^I, Eq. (3.19), we should take them into account first. The constraints are

$$\begin{aligned} \Phi^I = & P^I + \frac{2i}{\sqrt{2p^+}}\left((p^+ - hP^-)\delta^{IJ} + \frac{1}{2}hX'^- s^{IJ} \right)\psi^J \\ & \approx 0 + \mathcal{O}(\psi^3), \end{aligned} \tag{3.22}$$

where the rescaling (3.21) has been performed. Here P^- and X'^- should be understood as the solutions of the Virasoro constraints (3.16). Then by using the canonical Poisson brackets

$$\begin{aligned} \{X^i(\sigma), P^j(\sigma')\}_{PB} &= \delta^{ij}\delta(\sigma - \sigma'), \\ \{\psi^{I\alpha}(\sigma), P^{J\beta}(\sigma')\}_{PB} &= -\delta^{IJ}\delta^{\alpha\beta}\delta(\sigma - \sigma'), \end{aligned} \tag{3.23}$$

one can show that Φ^I are in second class and their time evolutions give no more constraints. The usual Dirac procedure for the constrained system [26] then leads to the Dirac bracket, $\{,\}_D$, consistent with the constraints (3.22). The resulting nonvanishing Dirac brackets are

$$\begin{aligned} & \{X^i(\sigma), P^j(\sigma')\}_D \\ & \quad = \delta^{ij}\delta(\sigma - \sigma') + \mathcal{O}(\psi^4), \\ & \{\psi^{1\alpha}(\sigma), \psi^{1\beta}(\sigma')\}_D \\ & \quad = -i\left(1 - \frac{h}{4(p^+)^2}(P^i - X'^i)^2\right)\delta^{\alpha\beta}\delta(\sigma - \sigma') + \mathcal{O}(\psi^2), \\ & \{\psi^{2\alpha}(\sigma), \psi^{2\beta}(\sigma')\}_D \\ & \quad = -i\left(1 - \frac{h}{4(p^+)^2}(P^i + X'^i)^2\right)\delta^{\alpha\beta}\delta(\sigma - \sigma') + \mathcal{O}(\psi^2), \\ & \{X^i(\sigma), \psi^{I\alpha}(\sigma')\}_D \\ & \quad = -\frac{h}{4(p^+)^2}(P^i\delta^{IJ} - X'^i s^{IJ})\psi^{J\alpha}\delta(\sigma - \sigma') + \mathcal{O}(\psi^3), \\ & \{P^i(\sigma), \psi^{1\alpha}(\sigma')\}_D \\ & \quad = \frac{1}{8(p^+)^2}\partial_i h(P^j - X'^j)^2\psi^{1\alpha}\delta(\sigma - \sigma') \\ & \qquad + \frac{1}{4(p^+)^2}[h(P^i - X'^i)\psi^{1\alpha}](\sigma')\partial_\sigma\delta(\sigma - \sigma') \\ & \qquad + \mathcal{O}(\psi^3), \\ & \{P^i(\sigma), \psi^{2\alpha}(\sigma')\}_D \\ & \quad = \frac{1}{8(p^+)^2}\partial_i h(P^j + X'^j)^2\psi^{2\alpha}\delta(\sigma - \sigma') \\ & \qquad - \frac{1}{4(p^+)^2}[h(P^i + X'^i)\psi^{2\alpha}](\sigma')\partial_\sigma\delta(\sigma - \sigma') \\ & \qquad + \mathcal{O}(\psi^3). \end{aligned} \tag{3.24}$$

For the Dirac brackets between X^i and P^i, we have omitted the terms of order $\mathcal{O}(\psi^2)$ since they are of order $\mathcal{O}(h^2)$. The terms of quadratic order in ψ in the Dirac brackets between ψ's have not been given due to the fact that, in order to determine them, we need to know the terms of order $\mathcal{O}(\psi^3)$ in the constraints (3.22) which are supposed to come from the not yet determined $\mathcal{O}(\psi^4)$ terms in the Hamiltonian (3.20).

Having the Dirac brackets, through the time evolutions $\dot{X}^i=\{X^i,H\}_D$ and $\dot{\psi}^I=\{\psi,H\}_D$, we now get the equations of motion for X^i and ψ^I as follows:

$$\begin{aligned}\dot{X}^i=&\frac{1}{p^+}P^i-\frac{h}{2(p^+)^3}[(P^j)^2+(X'^j)^2]P_i+\frac{h}{(p^+)^3}(P^jX'^j)X'^i\\&-i\frac{h}{2(p^+)^3}(P^i-X'^i)\psi^1\psi'^1+i\frac{h}{2(p^+)^3}(P^i+X'^i)\\&\times\psi^2\psi'^2+\frac{i}{16(p^+)^3}\partial_jh(P^k-X'^k)^2\psi^1\gamma^{ji}\psi^1\\&+\frac{i}{8(p^+)^3}\partial_jh(P^i-X'^i)(P^k+X'^k)\psi^1\gamma^{jk}\psi^1\\&+\frac{i}{16(p^+)^3}\partial_jh(P^k+X'^k)^2\psi^2\gamma^{ji}\psi^2+\frac{i}{8(p^+)^3}\partial_jh\\&\times(P^i+X'^i)(P^k-X'^k)\psi^2\gamma^{jk}\psi^2+\mathcal{O}(\psi^4),\end{aligned}\tag{3.25}$$

$$\begin{aligned}\dot{\psi}^1=&\frac{1}{p^+}\psi'^1-\frac{h}{2(p^+)^3}(P^i-X'^i)^2\psi'^1-\frac{1}{8(p^+)^3}\partial_ih\\&\times(P^j-X'^j)^2(P^i+X'^i)\psi^1+\frac{1}{8(p^+)^3}\partial_ih(P^k-X'^k)^2\\&\times(P^j+X'^j)\gamma^{ij}\psi^1+\mathcal{O}(\psi^3),\\
\dot{\psi}^2=&-\frac{1}{p^+}\psi'^2+\frac{h}{2(p^+)^3}(P^i+X'^i)^2\psi'^2-\frac{1}{8(p^+)^3}\partial_ih\\&\times(P^j+X'^j)^2(P^i-X'^i)\psi^2+\frac{1}{8(p^+)^3}\partial_ih(P^k+X'^k)^2\\&\times(P^j-X'^j)\gamma^{ij}\psi^2+\mathcal{O}(\psi^3).\end{aligned}\tag{3.26}$$

We would like to note that these equations of motion in the point particle limit, i.e., eliminating the terms involving the σ derivatives, agree with those of the 11-dimensional supergraviton [7] except for the transverse $SO(9)$ invariance rather than $SO(8)$.

IV. SUPERSYMMETRY IN LIGHT-CONE GAUGE

The system described by the light-cone gauge Hamiltonian (3.20) is supersymmetric. In this section, we investigate the supersymmetry transformation rules for X^i and ψ^I, and the supersymmetry algebra. To begin with, we consider the supersymmetry preserved by the background geometry (3.2), which can be seen by looking at the Killing spinor equation coming from Eq. (2.10) in the background, $\delta_\eta\psi^I_\mu=0$:

$$D_\mu(\omega)\eta^I=0.$$

The solution of this equation shows that η^I is of the following form:

$$\eta^I=f^{-1/4}\begin{pmatrix}\epsilon^I\\ \epsilon^I\end{pmatrix},\tag{4.1}$$

where ϵ^I are the 16 component constant spinors. Since η^1 (η^2) has ten-dimensional positive (negative) chirality, ϵ^1 is in the representation $\mathbf{8_c}$ of spin (8) while ϵ^2 is in $\mathbf{8_s}$. Thus we see that the background geometry (3.2) preserves 16 supersymmetries in total. As shown in the case of the matrix theory [6], it is this abundance of supersymmetry that is responsible for the precise agreement between the SYM, matrix string theory, and the supergravity side calculations in the previous section.

Though they were enough to verify the invariance of superstring action in Sec. II A, the supersymmetry transformation rules (2.9) do not give us the full supersymmetry structure up to the quadratic order in terms of θ; we need the transformation rules expanded up to θ^2 order. They can be obtained from the results of Ref. [15] through the Kaluza-Klein reduction, and, for the background geometry (3.2), are given by

$$\begin{aligned}&\delta_\eta X^\mu=i\overline{\theta^I}\Gamma^\mu\eta^I+\mathcal{O}(\theta^3),\\&\delta_\eta\theta^I=\eta^I-\frac{i}{4}(\overline{\theta^J}\Gamma^\mu\eta^J)\omega_\mu^{rs}\Gamma_{rs}\theta^I+\mathcal{O}(\theta^4).\end{aligned}\tag{4.2}$$

In the light-cone gauge specified in Eqs. (3.6) and (3.17), these become

$$\begin{aligned}&\delta_\eta X^+=\delta_\eta X^-=0+\mathcal{O}(\theta^3),\quad \delta_\eta X^i=i\overline{\theta^I}\Gamma^i\eta^I+\mathcal{O}(\theta^3),\\&\delta_\eta\theta^I=\eta^I-\frac{i}{4}f^{-1}\partial_ih(\overline{\theta^J}\Gamma^-\eta^J)\Gamma^-\Gamma^i\theta^I\\&\qquad+\frac{i}{4}f^{-1}\partial_ih(\overline{\theta^J}\Gamma^i\eta^J)\theta^I+\mathcal{O}(\theta^4),\end{aligned}\tag{4.3}$$

with the supersymmetry parameter η^I given by Eq. (4.1) which satisfies $\Gamma^-\eta^I=0$. However, since $\Gamma^+\eta^I\neq0$, the above supersymmetry transformation rules break the κ-symmetry fixing condition [Eq. (3.6)], that is, $\Gamma^+\delta_\eta\theta^I\neq0$. This means that we should modify the above transformation rules for the correct supersymmetry in the light-cone gauge.[6] In order to preserve the κ-symmetry fixing condition, it is natural to use the κ transformations for the modification. We may also include the worldsheet diffeomorphism with parameter ζ in the modified supersymmetry transforma-

[6]This situation has been known in the study of GS superstring theory in a flat background. See, for example, Chap. 5 of Ref. [27].

tion rules, for the possibility of breakdown of the diffeomorphism fixing condition, the light-cone gauge, Eq. (3.17). Then the modified supersymmetry transformation δ is of the following form:

$$\delta=\delta_\eta+\delta_\kappa+\delta_\zeta, \tag{4.4}$$

where κ and ζ are the functions of η, i.e., ϵ, to be determined by the requirement of preserving the light-cone gauge.

The κ transformation rules, which also have an expansion in terms of θ are again obtained from the 11-dimensional results of Ref. [15] through the Kaluza-Klein reduction and, for the background (3.2), are given by

$$\delta_\kappa X^\mu = i\overline{\kappa^I_+}\Gamma^\mu\theta^I+\mathcal{O}(\theta^3),$$

$$\delta_\kappa\theta^I=\kappa^I_+ +\frac{i}{4}(\overline{\theta^J}\Gamma^\mu\kappa^J_+)\omega^{rs}_\mu\Gamma_{rs}\theta^I+\mathcal{O}(\theta^4), \tag{4.5}$$

where κ_+ is defined in Eq. (2.20). In the light-cone gauge, Eqs. (3.6) and (3.17), we get

$$\delta_\kappa X^+ = -if^{-1/2}h\overline{\kappa^I_+}\Gamma^-\theta^I+\mathcal{O}(\theta^3),$$

$$\delta_\kappa X^- = if^{-1/2}\overline{\kappa^I_+}\Gamma^-\theta^I+\mathcal{O}(\theta^3),$$

$$\delta_\kappa X^i = i\overline{\kappa^I_+}\Gamma^i\theta^I+\mathcal{O}(\theta^3),$$

$$\delta_\kappa\theta^I=\kappa^I_+ +\frac{i}{4f}\partial_i h(\overline{\theta^J}\Gamma^-\kappa^J_+)\Gamma^-\Gamma^i\theta^I - \frac{i}{4f}\partial_i h(\overline{\theta^J}\Gamma^i\kappa^J_+)\theta^I+\mathcal{O}(\theta^4). \tag{4.6}$$

For the superstring case, it is usually convenient to introduce

$$\kappa^I_m \equiv -i\frac{\sqrt{-\gamma}}{2\sqrt{-g}}\Pi_m\Gamma\kappa^I, \tag{4.7}$$

which allows us to view the transformation parameter κ as a worldsheet vector. By using Eqs. (2.21) and (3.10), it is easy to show that κ^1_m (κ^2_m) satisfies the (anti-) self-dual condition:

$$\kappa^{1m}=P^{mn,11}\kappa^1_n, \quad \kappa^{2m}=P^{mn,22}\kappa^2_n. \tag{4.8}$$

Therefore, each of these worldsheet vectors has one independent vector component and hence may be represented as[7]

$$\kappa^{Im}=2P^{m\tau,IJ}\chi^J. \tag{4.9}$$

We now turn to the modified supersymmetry transformation, Eq. (4.4), and investigate it order by order in terms of the anticommuting coordinates. At the leading order, we first consider the transformation of θ,

$$\delta^{(0)}\theta^I=\delta^{(0)}_\eta\theta^I+\delta^{(0)}_\kappa\theta^I+\zeta^{(0)m}\partial_m\theta^I, \tag{4.10}$$

where the superscript (n) represents that the explicit order of θ (i.e., ψ) is n. To preserve the κ-symmetry fixing condition, this must satisfy

$$\Gamma^+\delta^{(0)}\theta^I=\Gamma^+(\delta^{(0)}_\eta\theta^I+\delta^{(0)}_\kappa\theta^I)=0. \tag{4.11}$$

We see that the diffeomorphism parameter $\zeta^{(0)m}$, which is zeroth order in θ, does not contribute to this consistency requirement and may be set to zero. On rewriting Eqs. (4.10) and (4.11) in terms of χ^I through Eqs. (4.7) and (4.9), one can show without much difficulty that $\delta^{(0)}\theta^I$ is given only in terms of η^I. If we now express the resulting transformation $\delta^{(0)}\theta^I$ in terms of the 16 component spinors ψ^I and ϵ^I by using Eqs. (3.8) and (4.1), we then have

$$\delta^{(0)}\psi^1=f^{-1/4}N^{1i}\gamma^i\epsilon^1,$$

$$\delta^{(0)}\psi^2=f^{-1/4}N^{2i}\gamma^i\epsilon^2, \tag{4.12}$$

where we have defined

$$N^{Ii}\equiv\frac{\gamma^{\tau\tau}\Pi^i_\tau+(\gamma^{\tau\sigma}\mp 1/\sqrt{-\gamma})\Pi^i_\sigma}{\gamma^{\tau\tau}\Pi^+_\tau+(\gamma^{\tau\sigma}\mp 1/\sqrt{-\gamma})\Pi^+_\sigma}, \tag{4.13}$$

with $\mp$ corresponding to $I=1,2$ respectively. The above transformation rules are the desired results that preserve the κ-symmetry fixing condition. Here we would like to note that, in the process of calculation, it is crucial to recognize the following identity satisfied for each I:

$$N^{Ii}N^{Ii}=-\frac{\gamma^{\tau\tau}\Pi^-_\tau+(\gamma^{\tau\sigma}\mp 1/\sqrt{-\gamma})\Pi^-_\sigma}{\gamma^{\tau\tau}\Pi^+_\tau+(\gamma^{\tau\sigma}\mp 1/\sqrt{-\gamma})\Pi^+_\sigma}. \tag{4.14}$$

In fact, the identity (4.14) is nothing but the covariant Virasoro constraint as can be verified by a direct calculation. At this point, one may worry about the presence of the worldsheet metric in N^{Ii}, about which we have not been concerned so far. However, as we shall see later, an intriguing fact for N^{Ii} is that, though the worldsheet metric appears in its definition, the resulting expression of N^{Ii} is totally independent on the worldsheet metric.

Let us now consider the leading-order modified supersymmetry transformation of X^i, $\delta^{(1)}X^i$. In this case, there is a problem related to the light-cone gauge $X^+=2\tau$, Eq. (3.17). The transformation $\delta^{(1)}X^+$ is given by

$$\delta^{(1)}X^+=4if^{-1/2}h(\psi^I\delta^{(0)}\psi^I)+2\zeta^{(1)\tau}, \tag{4.15}$$

which does not vanish in general and breaks the light-cone gauge fixing condition. In order to recover the light-cone gauge, $\zeta^{(1)\tau}$ should be chosen such as

$$\zeta^{(1)\tau}=-2if^{-1/2}h(\psi^I\delta^{(0)}\psi^I). \tag{4.16}$$

[7]Detailed expressions of κ^{Im} in terms of χ^I are as follows:

$$\kappa^{1\tau}=\gamma^{\tau\tau}\chi^1, \quad \kappa^{1\sigma}=(\gamma^{\tau\sigma}-1/\sqrt{-\gamma})\chi^1,$$

$$\kappa^{2\tau}=\gamma^{\tau\tau}\chi^2, \quad \kappa^{2\sigma}=(\gamma^{\tau\sigma}+1/\sqrt{-\gamma})\chi^2.$$

Then the transformation $\delta^{(1)}X^i$ becomes, up to diffeomorphism in the σ direction,

$$\delta^{(1)}X^i = 4if^{-1/4}\psi^I\gamma^i\epsilon^I - 2if^{-1/2}h(\psi^I\delta^{(0)}\psi^I)\dot{X}^i + \zeta^{(1)\sigma}X'^i. \tag{4.17}$$

Since the spatial component $\zeta^{(1)\sigma}$ does not appear in $\delta^{(1)}X^+$ due to $X'^+=0$ in the light-cone gauge, it remains undetermined. The requirement of preserving another condition in the light-cone gauge, Eq. (3.17), is not helpful for specifying it, since p^+ is constant. As a possible way of determining it, we consider the closure of supersymmetry algebra, a property that supersymmetry must satisfy. If we use the leading-order transformation rules for ψ^I and X^i, Eqs. (4.12) and (4.17), and the equation of motion for X^i, Eq. (3.25), the requirement of $[\delta_{\epsilon_{(1)}}, \delta_{\epsilon_{(2)}}]X^i = \xi^m\partial_m X^i$ with ξ^m as bilinear combinations of $\epsilon_{(1)}$ and $\epsilon_{(2)}$, makes us get

$$\zeta^{(1)\sigma} = 2i(p^+)^{-1}f^{-1/2}hs^{IJ}(\psi^I\delta^{(0)}\psi^J). \tag{4.18}$$

Through the same procedure performed at the leading order, we can obtain the next-to-leading-order corrections to the modified supersymmetry transformation. At this order, there is no need to consider the corrections to the transformation of X^i, since the next-to-leading-order corrections are of order $\mathcal{O}(\psi^3)$ and thus beyond our interest in this paper. The modified supersymmetry transformations of ψ^I get nontrivial corrections at the next-to-leading order and they are obtained as

$$\begin{aligned}\delta^{(2)}\psi^1 &= if^{-5/4}\partial_i h(\psi^I\gamma^i\epsilon^I)\psi^1 \\ &\quad - 2if^{-5/4}(\psi^I\delta^{(0)}\psi^I)\partial_i hN^{1j}\gamma^j\gamma^i\psi^1 + \zeta^{(1)m}\partial_m\psi^1, \\ \delta^{(2)}\psi^2 &= if^{-5/4}\partial_i h(\psi^I\gamma^i\epsilon^I)\psi^2 \\ &\quad - 2if^{-5/4}(\psi^I\delta^{(0)}\psi^I)\partial_i hN^{2j}\gamma^j\gamma^i\psi^2 + \zeta^{(1)m}\partial_m\psi^2,\end{aligned} \tag{4.19}$$

where $\zeta^{(1)m}$ are given by Eqs. (4.16) and (4.18).

Up to the quadratic order in ψ, we have obtained all the information for the supersymmetry transformations in the light-cone gauge. By gathering the order by order results, the full modified supersymmetry transformation rules preserving the light-cone gauge and κ-symmetry fixing conditions, are given by

$$\begin{aligned}\delta X^i &= \delta^{(1)}X^i + \mathcal{O}(\psi^3), \\ \delta\psi^I &= \delta^{(0)}\psi^I + \delta^{(2)}\psi^I + \mathcal{O}(\psi^4).\end{aligned} \tag{4.20}$$

The detailed form of the above transformations are obtained by doing the rescalings for ψ^I, Eqs. (3.11) and (3.21), using the equations of motion for X^i and ψ^I, Eqs. (3.25) and (3.26), and the following expansions for N^{Ii}, Eq. (4.13):

$$\begin{aligned}f^{-1/2}N^{1i} &= \frac{1}{2p^+}(P^i + X'^i) - \frac{1}{8(p^+)^3}(P^j - X'^j)^2(P^i + X'^i) \\ &\quad + \frac{i}{4(p^+)^3}h(P^i + X'^i)(\psi^2\psi'^2) + \frac{i}{8(p^+)^3} \\ &\quad \times \partial_j hP^kX'^k(\psi^1\gamma^{ij}\psi^1) + \frac{i}{16(p^+)^3}\partial_j h(P^i + X'^i) \\ &\quad \times (P^k\delta^{IJ} + X'^ks^{IJ})(\psi^I\gamma^{jk}\psi^J) + \mathcal{O}(\psi^4),\end{aligned}$$

$$\begin{aligned}f^{-1/2}N^{2i} &= \frac{1}{2p^+}(P^i - X'^i) - \frac{1}{8(p^+)^3}(P^j + X'^j)^2(P^i - X'^i) \\ &\quad - \frac{i}{4(p^+)^3}h(P^i - X'^i)(\psi^1\psi'^1) - \frac{i}{8(p^+)^3} \\ &\quad \times \partial_j hP^kX'^k(\psi^2\gamma^{ij}\psi^2) + \frac{i}{16(p^+)^3}\partial_j h(P^i - X'^i) \\ &\quad \times (P^k\delta^{IJ} + X'^ks^{IJ})(\psi^I\gamma^{jk}\psi^J) + \mathcal{O}(\psi^4).\end{aligned} \tag{4.21}$$

As alluded to before, we see that N^{Ii} does not have the dependence on the worldsheet metric. The final expressions are

$$\begin{aligned}\delta X^i &= \frac{2i}{\sqrt{2p^+}}\psi^I\gamma^i\epsilon^I - \frac{i}{2\sqrt{2p^+}(p^+)^2}h(P^i - X'^i)(P^j + X'^j) \\ &\quad \times (\psi^1\gamma^j\epsilon^1) - \frac{i}{2\sqrt{2p^+}(p^+)^2}h(P^i + X'^i)(P^j - X'^j) \\ &\quad \times (\psi^2\gamma^j\epsilon^2) + \mathcal{O}(\psi^3),\end{aligned}$$

$$\begin{aligned}\delta\psi^1 &= \frac{2}{\sqrt{2p^+}}(P^i + X'^i)\gamma^i\epsilon^1 - \frac{1}{2\sqrt{2p^+}(p^+)^2}h(P^j - X'^j)^2 \\ &\quad \times (P^i + X'^i)\gamma^i\epsilon^1 + \frac{i}{\sqrt{2p^+}(p^+)^2}h(P^i + X'^i) \\ &\quad \times (\psi^2\psi'^2)\gamma^i\epsilon^1 - \frac{i}{\sqrt{2p^+}(p^+)^2}h(P^i - X'^i) \\ &\quad \times (\psi^2\gamma^i\epsilon^2)\psi'^1 - \frac{i}{2\sqrt{2p^+}(p^+)^2}\partial_i h(P^kX'^k) \\ &\quad \times (\psi^1\gamma^{ij}\psi^1)\gamma^j\epsilon^1 + \frac{i}{4\sqrt{2p^+}(p^+)^2}\partial_i h(P^j + X'^j) \\ &\quad \times (P^k\delta^{IJ} + X'^ks^{IJ})(\psi^I\gamma^{ik}\psi^J)\gamma^j\epsilon^1 \\ &\quad - \frac{i}{4\sqrt{2p^+}(p^+)^2}\partial_i h(P^j + X'^j)(P^k\delta^{IJ} + X'^ks^{IJ}) \\ &\quad \times (\psi^I\gamma^k\epsilon^J)\gamma^j\gamma^i\psi^1 + \mathcal{O}(\psi^4),\end{aligned}$$

$$\delta\psi^2 = \frac{2}{\sqrt{2p^+}}(P^i - X'^i)\gamma^i\epsilon^2 - \frac{1}{2\sqrt{2p^+}(p^+)^2}h(P^j + X'^j)^2$$
$$\times(P^i - X'^i)\gamma^i\epsilon^2 - \frac{i}{\sqrt{2p^+}(p^+)^2}h(P^i - X'^i)$$
$$\times(\psi^1\psi'^1)\gamma^i\epsilon^2 + \frac{i}{\sqrt{2p^+}(p^+)^2}h(P^i + X'^i)$$
$$\times(\psi^1\gamma^i\epsilon^1)\psi'^2 + \frac{i}{2\sqrt{2p^+}(p^+)^2}\partial_i h(P^k X'^k)$$
$$\times(\psi^2\gamma^{ij}\psi^2)\gamma^j\epsilon^2 + \frac{i}{4\sqrt{2p^+}(p^+)^2}\partial_i h(P^j - X'^j)$$
$$\times(P^k\partial^{IJ} + X'^k s^{IJ})(\psi^I\gamma^{ik}\psi^J)\gamma^j\epsilon^2$$
$$-\frac{i}{4\sqrt{2p^+}(p^+)^2}\partial_i h(P^j - X'^j)(P^k\delta^{IJ} + X'^k s^{IJ})$$
$$\times(\psi^I\gamma^k e^J)\gamma^j\gamma^i\psi^2 + \mathcal{O}(\psi^4). \tag{4.22}$$

Obviously, these supersymmetry transformations are global from the worldsheet point of view, since the transformation parameters ϵ^I are constants, while the starting supersymmetry transformations (4.3) are local in target spacetime. If we set $h=0$ for a moment, the transformations, Eq. (4.22), are nothing but those of scalar multiplets of two-dimensional $\mathcal{N}=(8,8)$ SYM theory, obtained in the early days of GS superstring theory [28]. The interpretation of this is clear in the context of DLCQ M theory, though it is not so in the original GS superstring theory itself. The transformation rules for the case of $h=0$ are those of matrix string theory at the tree level corresponding to a free superstring. The h dependent terms in Eq. (4.22) are due to the one-loop corrections to the matrix string theory for the two superstring background.

With the modified supersymmetry transformation rules (4.22), it is a straightforward task to investigate the supersymmetry algebra. Since we have not determined the terms of the order $\mathcal{O}(\psi^3)$ in δX^i, it is not possible to check the supersymmetry algebra for X^i to quadratic order in terms of ψ. On the contrary, $\delta\psi^I$ leads to the nontrivial check. By using the $SO(8)$ Fierz identity for the spinors with the same $SO(1,9)$ chiralities,

$$(\psi\gamma^i\epsilon_{(1)})\gamma^i\epsilon_{(2)} = (\epsilon_{(1)}\epsilon_{(2)})\psi - \tfrac{1}{4}(\epsilon_{(1)}\gamma^{ij}\epsilon_{(2)})\gamma^{ij}\psi, \tag{4.23}$$

we can show that, up to the equations of motion,

$$[\delta_{\epsilon_{(1)}}, \delta_{\epsilon_{(2)}}]\psi^I = \xi^m\partial_m\psi^I + \mathcal{O}(\psi^3), \tag{4.24}$$

where the worldsheet translation parameters ξ^m are given by

$$\xi^\tau = 4i(\epsilon^1_{(1)}\epsilon^1_{(2)} + \epsilon^2_{(1)}\epsilon^2_{(2)}),$$
$$\xi^\sigma = 4i(p^+)^{-1}(\epsilon^1_{(1)}\epsilon^1_{(2)} - \epsilon^2_{(1)}\epsilon^2_{(2)}). \tag{4.25}$$

The algebra (4.24) is what we want in the two-dimensional theory and corresponds to the anticommutation relation between the would-be supercharges Q^I generating the transformations (4.22); $\{Q^I,Q^J\}\propto\delta^{IJ}H + s^{IJ}P$, where H and P are the translation generators in two dimensions. Besides the algebra (4.24), another thing to be investigated is the supersymmetry transformation property of the system described by the Hamiltonian (3.20), which is obtained as

$$\delta_{\epsilon^I}H = 0 + \mathcal{O}(\psi^3). \tag{4.26}$$

As usual, this means that the system is supersymmetric. In other words, the supercharges are conserved: $[Q^I,H]=0$.

V. DISCUSSION

We have studied the light-cone superstring dynamics on the gravitational wave background corresponding to the matrix string theory and investigated the structure of supersymmetry. This is the supergravity side analysis of the matrix string theory. Basically, because enough of supersymmetries preserved by the background, 16 supersymmetries, the results on dynamics have agreed with those obtained from the matrix string theory, the SYM side. The supersymmetry transformation rules in the light-cone gauge have been identified with those of the low-energy one-loop effective action of matrix string theory for two superstring backgrounds in weak string coupling. The importance of our results is that the supersymmetry transformation rules obtained in this paper may provide an alternative approach to determine some parts of the higher loop corrections to the low-energy effective action of matrix string theory without explicit loop calculations. The full expansion of transformation rules, up to 16th order in terms of the anticommuting coordinates ψ^I, will give us more information about the dynamics of matrix string theory or the light-cone superstring on the gravitational wave background.

In our formulation of the light-cone superstring, we have fixed two worldsheet diffeomorphisms by choosing two phase-space variables X^+ and P^+, the light-cone gauge (3.17). An intriguing fact is that the worldsheet metric has not appeared in the various final results, and hence we have not needed to worry about how it is fixed according to the light-cone gauge. If it had played a role in any way, our formulation would be quite complicated. It is hard to believe that this situation is an accident. We expect that the independence on the worldsheet metric also holds in other supergravity backgrounds, at least as far as the same kind of calculations done in this paper are concerned.

For supersymmetry in the light-cone gauge, we have not tried to get the conserved supercharges Q^I after obtaining the supersymmetry transformation rules (4.22). This is because Q^I generating Eq. (4.22) should have an expansion up to the order $\mathcal{O}(\psi^3)$, which requires knowledge about the terms of the order $\mathcal{O}(\psi^4)$ in the light-cone Hamiltonian (3.20). (Recall that the terms of that order in the light-cone Hamiltonian have not yet been determined.) Furthermore, since the Dirac brackets (3.24) are used to obtain the supersymmetry algebra and the transformation rules, i.e., $\delta = i\epsilon^I\{Q^I,\ \}_D$, we should know the terms of the order $\mathcal{O}(\psi^3)$ in the constraints (3.22) to get the Dirac brackets with the expansion up to the required order. For the case of the matrix theory, one of the present authors [7] has pointed out that the supercharges do not receive corrections of qubic or possibly higher order in anticommuting coordinates and have the same form as that for the flat case, while the Dirac brackets get corrections. If we believe that this is also the case in the present paper, the supercharges Q^I are simply given by

$$Q^I=\sqrt{2}(p^+)^{-1/2}\int d\sigma(P^i\delta^{IJ}+X'^i s^{IJ})\gamma^i\psi^J.$$

Indeed, if the Dirac brackets (3.24) are used, these supercharges generate the leading-order terms of the light-cone supersymmetry transformation rules [Eqs. (4.12) and (4.17)], i.e., $\delta^{(0)}\psi^I$ and $\delta^{(1)}X^i$. It is expected that the full transformation rules of Eq. (4.22) would be generated if the desired corrections were included in the Dirac brackets (3.24).

The final point we would like to discuss is that the formulation given in this paper is essentially perturbative in view of the matrix string theory. Off the conformal point, the matrix string theory has the electric sector that describes the dynamics of $D0$-branes. In Ref. [25], the process of exchanging $D0$-branes between two superstrings in the transverse direction has been considered. It is basically the instantonlike process, and is thus nonperturbative. On the supergravity side, how to see this process and, more generally, how to extend the present formulation to the off conformal regime of the matrix string theory, remains an interesting problem.

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